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ON THE EVALUATION OF TELECOMMUNICATIONS NETWORK RELIABILITY USING ROUTING MODELS

by

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ABSTRACT

We propose a measure of telecommunications network reliability based on the expected value of the lost call traffic due to failures in the network. The evaluation of the lost call traffic in the failed network will be performed by solving re-routing problems. The first formulation to be presented for the re-routing problem is a multicommodity model with explicit capacity constraints. Algorithms to solve this kind of problem are extremely time consuming. We then propose a new formulation for which the problem is modeled as a multicommodity one with nonlinear costs and without capacity constraints. The objective estimating the lost call traffic has been chosen to take into account the stochastic character of the demand. This new model solves the problem more easily and represents an approach closer to reality since stochastic demand is considered. An existing convex-simplex algorithm is used to solve problems of different sizes.

Key words : reliability, telecommunications network, routing, multicommodity problems.

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RÉSUMÉ

Nous proposons une mesure de fiabilité de réseaux de télécommunications basée sur l'espérance du traffic perdu dû aux pannes. L'évaluation de cette mesure est réalisée en résolvant des problèemes de routage d'appels. Nous présentons plusieurs formulations pour résoudre ces problèmes; la première est constituée par un modèle de multiproduit avec des contraintes de capacité. Nous montrons, alors, comment re-formuler le modèle pour qu'il n'y ait pas de contraintes de capacité tout en tenant compte du caractère stochastique de la demande. Ce nouveau modèle est à la fois plus facile à résoudre et plus proche de la réalité. Nous utilisons un algorithme de Simplexe Convexe pour résoudre plusieurs problèmes de dimensions différentes.

Mots-clés : fiabilité, réseau de télécommunications, routage, multiproduit.

1. INTRODUCTION

Reliability in telecommunications networks is a measure which is both hard to define and to evaluate. Traditionally network reliability has been defined as a connectivity measure of some kind [1,2,3]. The problem of evaluating some of the more popular connectivity measures of reliability in a network has been found to be NP-hard [4]. The situation is particularly difficult in a telecommunication network where the number of elements subject to failure is high and where failures of adjacent links can be dependent. Even if finding the exact value of a measure of connectivity is feasible, one has to ask if that kind of measure is a good reflection of the performance of the network. In fact, a telecommunication network is generally well connected even after failure of some of the elements. Therefore probabilities of different measures of connectivity are generally high so it is very difficult to distinguish a better performance from a standard one just by measuring connectivity.

We believe that in measuring the performance of the network it is imperative to take into account the following:

a) location of the failure;

b) capacity of the failed links;

c) importance of the lost calls.

In this paper, we propose a measure of reliability that shows the performance of the telecommunications network by taking into account these ideas and we find a method to evaluate it in a reasonable time.

2. METHODOLOGY

We have found that a good measure of reliability in a telecommunications network is related to the expected lost call traffic due to failures in the network. This measure can be interpreted as the capacity of the network to route calls even in the event of failures. In contrast with the

traditional connectivity measures, we could have a high connectivity network that, however, will present a large value of lost call traffic in the presence of failures. In order to evaluate the proposed measure, let us introduce our network.

A telecommunications network contains switching stations and transmission links. This can be formalized into a graph G(N,M), where N, the set of nodes, is the collection of switching stations, and M, the set of arcs, is the collection of transmission links. We suppose that only arcs are apt to fail. This is not necessarily true since nodes representing switching centers are likely to fail as well. However, if the failure of nodes were to be included in the model, failed nodes as well as their adjacent links should be removed. For simplicity, therefore, we prefer to make the assumption that only arcs can fail.

Another assumption often made in telecommunication network reliability analyses is the independence of link failures. This second hypothesis has been the object of controversy since adjacent logical links often share the same equipment and consequently their failures should be considered dependent. We have bypassed this difficulty by working with the physical instead of the logical network and by assuming that physical link failures are independent.

We now present the procedure to find our reliability measure for a circuit switched telecommunication network.

1) Find the weighted lost call traffic in the network when no failures are present. A weight can be arbitrarily given depending on the importance of the traffic blocked.

2) Find the expected lost call traffic in presence of failure in the network. This will be the object of subsequent discussion and constitutes the core of this work.

3) The difference between 1) and 2) will be a weighted measure of the expected lost call traffic due only to failure.

The value calculated in 3) constitutes the actual measure of reliability or unreliability of the network. The routing problem to be solved to evaluate point 1) will reappear to evaluate point 2) and will be discussed in sections 3 to 6. We are now going to explain in detail other necessary steps to evaluate point 2).

Suppose that links can be either failed or operating and that the state of the system is defined by the state of all its arcs. If m is the number of arcs in the network and S_k the representation of the k^{th} state of the system, we will need 2^m states to evaluate the exact value of the expectation of the lost call traffic in the system. Li and Silvester [5] have developed an algorithm that partially enumerates the t most probable states without analyzing all possible 2^m states. This methodology permits upper and lower bounds on the expectation of any measure of the system.

Denote $Z(S_k)$ the lost call traffic in the k^{th} state of the system. Then the upper and lower bounds on the expected lost call traffic can be obtained from the formulas given by Li and Silvester:

$$E(Z)_{up} = \sum_{k=1}^{t} P(S_k)\, Z(S_k) + \left(1 - \sum_{k=1}^{t} P(S_k)\right) Z(S_{2^m})$$

$$E(Z)_{low} = \sum_{k=1}^{t} P(S_k)\, Z(S_k) + \left(1 - \sum_{k=1}^{t} P(S_k)\right) Z(S_1) \qquad (1)$$

In (1) $P(S_k)$ represents the probability that the system is in the state S_k, $Z(S_1)$ is the lost call traffic when there are no failures in the system and $Z(S_2m)$ is the lost call traffic when all arcs have failed. We point out that $Z(S_1)$ is the result of 1). On the other hand when all arcs are down, the lost call traffic have to be equal to the total demand, so $Z(S_2m)$ = demand.

We are now going to resume in three steps the methodology to find the expected lost call traffic mentioned in point (2):

(2.1) Enumerate the t most probable states using the Li and Silvester algorithm.

(2.2) Evaluate the lost call traffic for every state enumerated in point (2.1) as well as for the state of total failure.

(2.3) Evaluate equation (1).

It is important to point out that when arcs availabilities are high, there is no need to use the Li and Silvester algorithm for partial state enumeration since the no-failure and single failure states are the most probable ones and cover most of the state space. This statement will be largely discuss in the Results section.

One of the differences between the Li and Silvester's approach and ours lay on the weight given to the enumeration procedure and on the way the performance measure is obtained for every state. The "goodness" of Li and Silvester's bounds is based on an effort to efficiently enumerate as many states as possible. On the other hand, since, as it will be later seen, most telecommunications networks have high arc availability, we consider that being very accurate in the lost call traffic evaluation for every state is more important than taking into account multiple failures states. Therefore, our performance measure will be based on the outcome of the routing models.

3. DETERMINISTIC ROUTING MODELS

In this section we will present the first kind of routing model. As it was stated before, our network reliability measure calculations are based on the routing problem. This, in terms of modelling and resolution, is the most difficult point to be solved in the proposed methodology.

Each state S_k represents a multicommodity network where traffic from every origin-destination pair constitutes a different kind of flow. The problem of evaluating the lost call traffic for a given state is an optimal routing one.

Our first model of routing can be classified as a classic formulation. The problem of finding the lost call traffic for a given state is transformed into a multicommodity problem where the commodities are the traffic of the same kind (for instance, the traffic for every origin-destination pair). In general, a multicommodity problem can be formulated as follows,

(Multicommodity Problem):

$$\boxed{\min Z}$$

subject to:

$$\boxed{\text{capacity constraints}}$$

$$\boxed{\text{flow conservation constraints}}$$

$$\boxed{\text{nonnegativity}}$$

Assad [6] has presented a survey on different approaches to solve the multicommodity problem. Kortanek and Polak [7] have discussed the design of telecommunication systems with demands variable per hour. Their model includes a routing subproblem formulated as a multicommodity problem. Our first model is somehow similar to the routing subproblem formulation of Kortanek and Polak.

We now define the necessary notation to formulate DM1 (First Deterministic Model).

Notation

x_u^{pq} traffic flow (in Erlang) origin-destination (origin = node p, destination = node q) using the arc u

C_u capacity of the arc u

d^{pq} origin-destination offered load

y^{pq} origin-destination lost traffic

$\Gamma(i)$ set of arcs whose origin node is i

$\Gamma^-(i)$ set of arcs whose terminal node is i

k^{pq} weighting given to the p-q commodity

O origin node set

D destination node set

N node set for the whole graph

M arc set for the whole graph

n cardinality of N

m cardinality of M

Then the DM1 formulation is:

$$\min Z = \sum_{p \in O} \sum_{q \in D} k^{pq} y^{pq} \qquad (2)$$

s.t.

Capacity:

$$\sum_{p\in O}\sum_{q\in D} x_u^{pq} \leq C_u \text{ , for all } u\in M \tag{3}$$

Flow conservation:

$$\sum_{u\in\Gamma(i)} x_u^{pq} - \sum_{u\in\Gamma^-(i)} x_u^{pq} = \begin{cases} d^{pq} - y^{pq}, & \text{if } i=p \quad \text{for all } p\in O \text{ ,} \\ -d^{pq} + y^{pq}, & \text{if } i=q \quad \text{for all } q\in D \text{ ,} \\ 0 \quad \text{otherwise} & \text{for all } i\in N \end{cases} \tag{4}$$

Nonnegativity:

$$x_u^{pq} \geq 0 \text{ , } y_{pq} \geq 0 \quad \text{for all } u\in M \text{ , } p\in O \text{ , } q\in D \text{ .} \tag{5}$$

The formulation DM1 proposed above gives rise to $O(n^3)$ constraints and $O(n^4)$ variables. In real networks $O(n)$ is not negligible so we reformulate DM1 in order to reduce by $O(n)$ the number of constraints and variables. This new approach is called DM2 (Second Deterministic Model).

Let:

x_u^p traffic flowing through arc u which origin is node p

D_p set of destination nodes for the traffic originating at node p

Then the DM2 formulation is:

$$\min Z = \sum_{p\in O}\sum_{q\in D_p} k^{pq} y^{pq} \tag{6}$$

s.t.

Capacity:

$$\sum_{p \in O} x_u^p \leq C_u \text{ , for all } u \in M \tag{7}$$

Flow conservation:

$$\sum_{u \in \Gamma(i)} x_u^p - \sum_{u \in \Gamma^-(i)} x_u^p = \begin{cases} \sum_{q \in D_p} d^{pq} - \sum_{q \in D_p} y^{pq}, & \text{if } i=p \\ -d^{pi} + y^{pi}, & \text{if } i \in D_p \\ 0 & \text{otherwise} \end{cases} \quad \begin{array}{l} \text{for all } p \in O \text{ ,} \\ \\ \text{for all } i \in N \end{array} \tag{8}$$

Nonnegativity:

$$x_u^p \geq 0 \text{ , } y^{pq} \geq 0 \quad \begin{array}{l} \text{for all } u \in M \text{ ,} \\ p \in O \text{ ,} \\ q \in D \text{ .} \end{array} \tag{9}$$

The DM1 and DM2 models have been formulated so it is possible for the origin-destination traffic to use any path in the network. However it is possible to add to the lost call traffic other terms in order to favour the shortest path and avoid over-loading unwanted links.

4. EXTENSIONS TO OBTAIN MORE REALISTIC MODELS

Modification to favour the shortest path

For the DM1 model the new objective will be:

min Z_1

where,

$$Z_1 = \sum_{p \in O} \sum_{q \in D} k^{pq} y^{pq} + \sum_{u \in M} \sum_{p \in O} \sum_{q \in D} \ell_u w_u x_u^{pq} \tag{10}$$

In (10) we have introduced the following notation:

ℓ_u length of the arc u

w_u weight given to arc u

Modifications to avoid blocking problems

The introduction of weights and arc lengths in the objective function aims at inducing the shortest path routing as well as the minimum lost calls for important customers. Those desirable features can in some cases give rise to unwanted re-routing options. We will now expose the problems which can be encountered.

Suppose that a failure occurs in a path used by a very important commodity. The program will try to re-route the important flow to other paths even if they are far away from the location of the failure. This can produce a blocking problem between nodes which are not been involved with the failure.

The situation described above is far from ideal and may be considered unacceptable. To remedy this kind of behavior we propose one of two strategies:

- reduce the network by forbidding certain commodities to pass through specific arcs;
- penalize the passage of unwanted commodities over certain arcs.

Both options are easy to implement in the DM1 formulation. However in DM2 it is not possible to affect the routing from an origin to a destination by network reduction or penalization without affecting the routing from the same origin to all possible destinations.

The choice of model depends on the characteristics of the network. If its size is big, DM2 is convenient since its formulation is O(n) smaller than DM1. However one has to consider that because of its completeness DM1 can present more subtleties than DM2.

5. STOCHASTIC ROUTING MODELS

Until now, we have considered the demand as a deterministic value. In reality, the demand is a stochastic variable. Kortanek and Polak [7] took this fact into account by considering that demands are Poisson and the flows through the links are Erlang difference distributed random variables. A penalty per unit out-of-service is added to the objective function. Capacity and flow conservation constraints are kept in this new model called SDR (Stochastic Dynamic Routing).

The models to be presented in this section take into account the random nature of the demand. The first one is called SM1 (First Stochastic Model). The asset of this model is to take advantage of the stochastic character of the demand not only to be closer to reality but to simplify the resolution of the problem. In fact, DM1 and DM2 are extremely difficult models to solve due to the capacity constraints related to the multicommodity flows. Usually this kind of problems are solved by decomposition procedures or by dropping the capacity constraint and adding a penalty function to the objective instead. In our case, we replace the capacity constraints by a penalty function that reflects the stochastic character of the demand and avoids, at the same time, the capacity of the arcs to be surpassed. This kind of approach has been successfully applied in air transportation (see Soumis [8], Lavoie [9]).

The flow is treated as a random variable with a probability function as the one shown in Figure 1. In this figure, the striped region corresponds to the probability that a unit of flow will be refused.

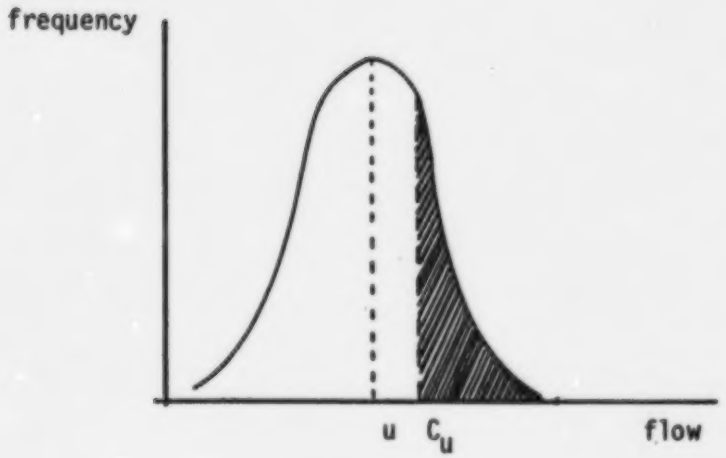

Fig. 1. Hypothetical probability distribution of the traffic flow on arc u.

Since it is difficult to deal with random variables in mathematical programming, we prefer to work with expected values instead of probability distributions and with averages instead of random variables. In order to minimize the lost call traffic, we will add a penalty function to the objective. This function can be the expected lost call traffic whose shape should be similar to the one displayed in Figure 2.

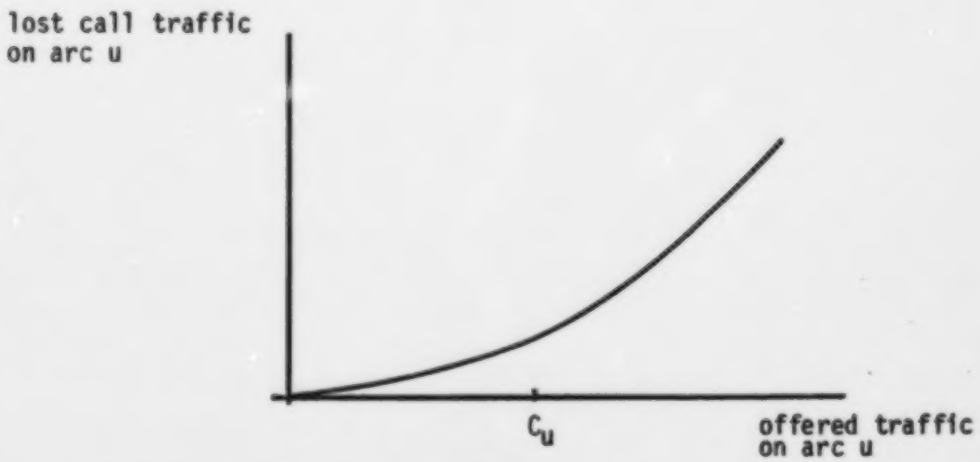

Fig. 2. Shape of the lost call traffic function.

We now have two options: either construct the function using real data or make the well known assumptions regarding the distribution of the lost calls and use the Erlang loss call formula (see [10]).

The Erlang loss formula, B(s,a) represents the proportion of lost calls in the long run or the probability of finding all lines busy:

$$B(s,a) = \frac{a^s/s!}{\sum_{k=0}^{s} a^k/k!}, \tag{11}$$

in (11) a is the average number of calls offered per holding time and s is the capacity of the trunk in number of lines.

From (11) it is possible to deduce the expected value of the lost traffic in a trunk as aB(s,a). Let x_u^{pq} be the offered traffic of the pq commodity on arc u, then the penalty function to be added to the objective will be:

$$P(X) = \sum_{u \in M} \sum_{p \in O} \sum_{q \in D} k^{pq} x_u^{pq} B(C_u, \sum_{p \in O} \sum_{q \in D} x_u^{pq}) \tag{12}$$

and the objective function Z_1 will be composed by a shortest path term and the penalty shown above:

$$Z_1 = L(X) + P(X) \tag{13}$$

where L(X) is the term that guarantees the shortest path and avoids blocking problems. Z_1 is a convex function since P is a convex function too. The convexity of P for a fixed capacity C_u and a variable demand was proved in [11], a different proof is shown in [12]. We now have all the elements to formulate our first stochastic model (SM1).

Formulation of SM1:

min L(X) + P(X)

s.t.

Flow conservation:

$$\sum_{u\in\Gamma(i)} x_u^{pq} - \sum_{u\in\Gamma^-(i)} x_u^{pq} = \begin{cases} d^{pq} & \text{, if } i=p \\ -d^{pq} & \text{, if } i=q \\ 0 & \text{otherwise} \end{cases} \qquad (14)$$

Nonnegativity:

$$x_u^{pq} \geq 0 \qquad (19)$$

In the model proposed above, d^{pq} is the average demand from node p to node q.

SM1 is in every way a better model compared to DM1 or DM2. Not only is the stochastic character of the demand taken into account, but the resolution of the problem has been considerably simplified by the elimination of the capacity constraint.

It should be noted that in SM1, the variable x_u^{pq} no longer represents the exact traffic flowing from p to q on arc u, but rather a traffic intensity which one tries to route through the arc. More precisely, (x_u^{pq}/d^{pq}) is the proportion of the calls from p to q which will be routed through arc u (subject to losing calls). One should also be aware that in this model the probability of losing a call routed on a path $u_1,u_2,\ldots,u_k$ is implicitly taken as being equal to $\sum_{j=1}^{k} B(C_{u_j}, \sum_{p\in O} \sum_{q\in D} x_{u_j}^{pq})$, while a more correct estimate, would be $[1 - \prod_{j=1}^{k} (1 - B(C_{u_j}, \sum_{p\in O} \sum_{q\in D} x_{u_j}^{pq}))]$ (this estimate is still somewhat erroneous, since it is based on the assumption that the actual flows carried by the links at any given point in time are independent random variables - which they are obviously not). In many communication networks however, the bias introduced by the simpler formula can be negligible, since the capacity of the links is large enough to make the probability of losing calls very small. In any case the simpler formula provides an upper bound on the blocking probability.

In order to reduce the number of variables we propose a similar model where commodities are defined as the traffic flowing from a node. We are going to use the same notation introduced for the DM2 model.

We now show the formulation for the second stochastic model (SM2):

$$\min L(X) + P(X) \tag{16}$$

s.t.

Flow conservation:

$$\sum_{u \in \Gamma(i)} x_u^p - \sum_{u \in \Gamma^-(i)} x_u^p = \begin{cases} \sum_{q \in D_p} d^{pq}, & \text{if } i=p \\ -d^{pi}, & \text{if } i \in D_p \\ 0 & \text{otherwise} \end{cases} \quad \begin{array}{l} \text{for all } p \in O, \\ \text{for all } i \in N \end{array} \tag{17}$$

Nonnegativity:

$$x_u^p \geq 0, \text{ for all } u \in M, \quad p \in O \tag{18}$$

$$\text{where } P(X) = \sum_{u \in M} \sum_{p \in O} k^p x_u^p B(C_u, \sum_{p \in O} x_u^p) \tag{19}$$

and $L(X)$ is a function that guarantees the shortest path and avoids blocking problems. In (19) k^p is a weighting given to commodity p.

6. STOCHASTIC ROUTING RESOLUTION

The model examined in this study, SM2, is a multicommodity problem with convex costs and without capacity constraints. If the costs were linear the problem would be solved very easily: it would only be necessary to solve a minimum cost optimal flow problem for each commodity. We propose a method to solve the nonlinear problem which makes as much use as possible of the separation of commodities.

The convex simplex algorithm can be adapted to take advantage of the network structure and the commodity separation. If the variables of the basis are regrouped by commodity each group becomes a spanning tree of the network. Because of this tree-structure no matrix storage is required for the simplex iterations. Furthermore the fact that we have as many trees as we have commodities will reduce even further the calculations; only the tree who belongs to the entering variable commodity will be used in one iteration. Finally, instead of selecting the entering variable to be that with the most negative reduced cost a different choice procedure is proposed (see Figure 3). The selection is made by exhausting for one commodity k all the variables whose reduced cost is inferior to ε_1 before studying another commodity. This is exactly as optimizing the commodity k with a termination criteria on ε_1 for the reduced costs. As we have mentioned before the fact of temporarily concentrating on just one commodity reduce computational effort and memory space since only the tree of the basis and the variables of the exploring commodity will be stored.

```
Set ε1 to some initial value
modif:=false;
while ε1< ε2 or modif:=true do
     begin
        modif:=false
        for k:=1 to K do
            begin
               Optimize commodity k as long as there are
               variables with reduced costs < 1;
               if some variables have changed,
                    set modif:=true
            end
        ε1:= ε1* λ .
     end
```

Fig. 3. Procedure to choose the entering variable in the adapted convex simplex algorithm.

Since the problem is a nonlinear one it will be necessary to optimize each commodity more than once to get at the global optimal solution. Therefore the process described above has to be repeated; ε_1 is gradually reduced by a rate $\lambda(\lambda<1)$ until the ε_2 level is reached. ε_2 is the chosen termination criteria. When a $\varepsilon_1 \leq \varepsilon_2$ is reached and no improved solution has been found for any commodity then we are at the desired global optimum. The gradual reduction of ε_1 will prevent the algorithm from spending too much time on one commodity if the others have not been suitably adjusted. It is interesting to point out that if λ is near 1 and ε_1 is very big at the beginning, the algorithm will behave as the simplex with the traditional entering cost criteria; the most negative cost variable will be selected and will change commodity very often. On the other hand if $\varepsilon_1 = \varepsilon_2$ the algorithm will behave as a decomposition, commodities will very seldom be changed . Therefore a good calibration of ε_1 and λ will allow to get an intermediate algorithm which will be more efficient that either of the extremes: tradional simplex or decomposition.

The adaptation of the simplex algorithm for multicommodity problems with convex costs was developed by M. Sauvé [13]. We have used this algorithm to test several problems.

We point out that even if the estimation of the reliability requires the solution of a routing problem for each state of the system, the calculation time are not multiplied by the number of states. In fact all the routing problems in the failed network are very similar to the problem when no failures are present. Therefore, once the no-failure routing is obtained the failure-present routing is easily found by re-optimization, starting the algorithm at the known no-failure optimal solution.

7. RESULTS

The first question that arises when we want to use our methodology to solve a real problem is how big does t, the number of states considered, have to be in order to cover a good proportion of the state space by partial enumeration.

The number of states to be considered depends on the arc availability of the network to be studied.

In the test networks provided by a telecommunications company, arc reliabilities were greater than 99.5 %. For such a level of availability, t, the number of states enumerated can be just m+1, that is, the single failure plus the no-failure states.

The previous statement is supported by figures 4 and 5. These figures represent the state space proportion covered by the no-failure and single failure states versus arc availability for networks with 60, 80 and 100 arcs. Figure 4 depicts the state space proportion where all arcs in the network have the same availability, which is greater than 90 %. Since we were interested in networks with arc availability greater than 99 %, we plotted in figure 5 the state space proportion for those availability values. From figure 5 it can be observed that when arc availability is 99.5 %, more than 93 % of the state space is covered by just enumerating no-failure and single failure states in a 60 arc network. These results confirm our decision, stated earlier, to put a lot of effort in the modelling and routing resolution and not to implement a special state enumeration procedure to take into account multiple failures states.

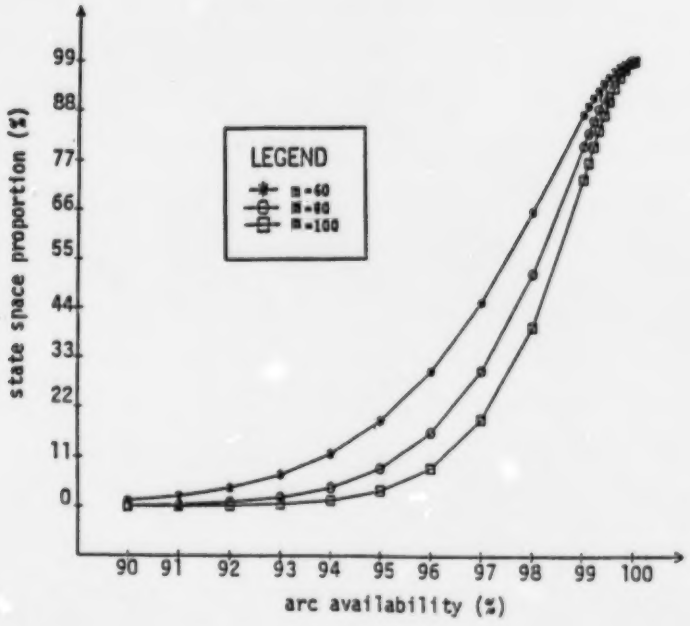

Fig. 4. State space proportion covered by the no-failure and single failure states vs arc availability.

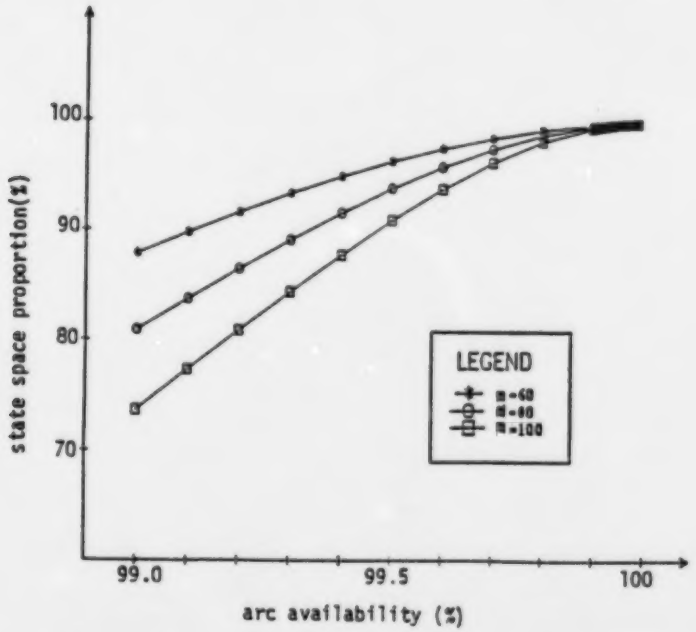

Fig. 5. Detail of figure 4 when arc availability is greater than 99 %.

The procedure to evaluate network performance was implemented with the SM2 routing model. Several networks were tested to study the variations of the resolution time versus network size, number of variables and number of states considered. The network provided by the telecommunications company was called RED. The data created for the smaller and bigger networks was created by taking into account the RED magnitude values. The tests were performed with the convex simplex algorithm described in section 6 and on a SUN 3/50.

Total execution times to solve SM2 when the no-failure and single failure states are considered are displayed in Table 1.

Table 1.

	RED 1	RED 2	RED	RED 3
Number of Nodes	10	20	40	60
Number of Arcs	14	33	62	110
O/D Pairs	100	400	1 600	3 600
Number of variables	280	1 320	4 960	13 200
Number of states considered	15	34	63	111
Total Execution Time (sec)	20.3	276.5	520.1	4 524.2

The results on Table 1 show that the proposed methodology gives good results. A small problem as RED 1, is solved in 20.3 sec. in a small machine (a SUN 3/50 is a 16.67 MHz computer). The standard problem, a

multicommodity routing with 40 commodities and 4 960 variables was solved 63 times in less than ten minutes. Even the results for the bigger problem are good, in fact we have solved 111 times a 60 commodity routing with 13 200 variables, in just about one hour and fifteen minutes.

8. CONCLUSIONS

We have proposed a flexible and easily computed measure of network reliability which realistically depicts the performance of the network under failure. The evaluation of this measure involves the resolution of a series of routing subproblems which may be formulated in different ways. We have suggested a new stochastic model for this routing subproblem. The main advantages of this new formulation are a more accurate assessment of the traffic which is lost due to failures and the low computing times, since it can be solved using a very efficient adaptation of the convex simplex algorithm.

We have however to point out that the methodology is useful if arc availability are very high, as is the case in most telecommunication networks. In fact, if arc availability were lower than 95 % for medium size networks, it would be necessary to enumerate multiple failures states for which a special enumeration algorithm should be implemented and the number of states to take into account could be tremendously high.

Regarding further work, we are currently working on other routing models which should be closer to the actual and forthcoming policies of telecommunications companies regarding re-routing of traffic subsequent to failures and on modifications of the methodology proposed in this paper to be used in network design.

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